MAXIMUMS AND MINIMUMS

Relative Maximums and Minimums:

A relative extrema (max or min) occurs when the slope of a function changes signs (positive/negative). A maximum occurs when the slope changes from positive to negative, while a minimum occurs when the slope changes from negative to positive. A “relative” max or min is also known as a “local” max or min.

When looking at a graph, the easiest way to find a max or min is to find the top or bottom of a curve. The top of a downward-facing curve is a relative maximum, while the bottom of an upward-facing curve is a relative minimum. An interval may have multiple curves, which produce multiple maximums and/or minimums.

Point b is a relative maximum

Point c is a relative minimum

A more specific way of finding a max or min is when the tangent line at a given point is completely horizontal. A tangent line is a line that intersects a curve exactly once. When graphing, a tangent line gives the instantaneous velocity of a (position) function at a given point. When the slope of a tangent line changes from increasing to decreasing or vice versa, a max or min is created in the function.

The tangent line is horizontal at points b and c. The slope of the tangent line changes from positive to negative at point b, then from negative to positive at point c, so point b is a maximum and point c is a minimum.

Another way to think of this is when the derivative (instantaneous velocity) changes signs. Mathematically, the easiest way to find a max or min of a function is to find the x-value where the derivative of the function is equal to zero. Then input points before and after that value to check if the derivative changes signs.

$f(x) = ?$ $f'(x) = 0$

$f'(b) = 0$

$f'(e) = +$; $f'(g) = -$

f(x) represents the normal function f'(x) represents the derivative of the function. Set the derivative equal to zero. the derivative is equal to zero at point x = b e represents a point on the interval (a,b), and the derivative at this point is positive. g represents a point on the interval (b,c), and the derivative is negative.

The derivative changes from positive to negative at x = b, so b is a maximum.

Absolute Maximums and Minimums:

An absolute max or min can only be found on a closed interval. In order to find an absolute max or min, all relative maximums and minimums must be found first. The actual value of the maximums and minimums must be found (f (x)). Next take the endpoints of the interval and find the values for each. Out of all the values found (endpoints and relative maximums and minimums), the point that has the highest value is the absolute maximum, while the point with the lowest value is the absolute minimum.

$f(b) = 3$ $f(c) = -3$

$f(a) = -2$ $f(d) = 5$

The absolute maximum is at x = d, because it has the highest value. Although x = b is a relative maximum, it is not an absolute maximum. x = c is both a relative and absolute minimum since it has the lowest value. x = a is neither a relative or absolute max or min.